Gap Wave Propagation in Functionally Graded Piezoelectric Material Structures

Jianke Du, Kai Xian, Ji Wang
Piezoelectric Device Laboratory, Department of Mechanics and Engineering Science, 818 Fenghua Road,
School of Engineering, Ningbo University, Ningbo, Zhejiang 315211, China
E-mail: dujianke@nbu.edu.cn

Abstract-Shear horizontal gap wave propagating between functionally graded piezoelectric material (FGPM) layer and a layered piezoelectric structure is investigated analytically. The electrically open conditions on strip surface are applied to solve this problem. The phase velocity can be numerically calculated for the electrically open case, with different thickness of the layer and wavenumber. The effect of the gradient variation about material on the phase velocity is discussed in detail. We find that gradient distributing of the material properties has remarkable effect on the phase velocity of the gap waves.

Ⅰ. INTRODUCTION

It is well known that gap waves can be utilized in piezoelectric devices for various applications, because of electric fields can exist in free space. Anti-plane piezoelectric gap waves in piezoceramic structure were studied in [1]-[2]. Piezoelectric gap waves between a piezoceramic half-space and a piezoceramic plate were investigated in [3]. A new-style material called functionally graded material (FGM) was proposed to solve problems in the thermal-protection systems of a space-plane in 1980s. From then on, FGM has attracted interest of investigators from many kinds of disciplines. Today, FGM can be used not only in thermal-protection systems but also in electronic and many other fields. The results obtained for the FGM layered structures lead us to consider that the FGM may be applicable to surface acoustic wave (SAW) devices.

Since White [4] invented the interdigital transducers utilized for transmitting and receiving SAW in 1965, SAW are applied successfully to electronic industry, such as filters, delay lines, resonators and oscillators etc. With the development of the material technology, FGPMs are manufactured and used in SAW devices to improve the efficiency and natural life of the SAW devices. Hence, the research of wave propagation behaviors and characteristics in FGPM has become a topic of practice importance. Liu and Tani investigated surface waves in FPGM plates with the application of strip element method [5-10]. Han et al. introduced a hybrid numerical method (HNM) to analyze characteristics of waves and transient responses in FGM cylinders [11-12]. Recently Han and Liu investigated the frequency and group velocity dispersion behaviors, and characteristic surfaces of waves in FGPM cylinders using an analytical numerical method [13]. Li et al. studied the behaviors of Love waves in a layered functionally graded piezoelectric structure using WKB method [14].

In this paper, shear horizontal gap wave propagation in a functionally graded piezoelectric strip and a layered piezoelectric structure is investigated analytically.

Ⅱ. PROBLEM FORMULATION

Consider a FGPM strip and an elastic substrate covered by a thin piezoelectric material layer illustrated in Fig.1. The piezoelectricity of the strip and the thin layer is polarized in z-axis direction. On the assumption that the SH waves propagate in the y direction, the total out-of-plane displacement and the electric potential are expressed as

$$u(x,y,t)=0\ ,\quad v(x,y,t)=0\ ,$$
$$w=w(x,y,t)\ ,\quad \phi=\phi(x,y,t)\ , \tag{1}$$

where u,v,w are the displacements and ϕ is the electric potential. The strains and the electric field can be given as

$$\gamma_{xz}=w,_x\ ,\quad \gamma_{yz}=w,_y\ , \tag{2}$$

$$E_x=-\phi,_x\ ,\quad E_y=-\phi,_y\ , \tag{3}$$

where γ_{ij} and E_i are the strains and electric field, respectively. The non-vanishing shear stresses can be given as

$$\begin{aligned}\tau_{xz}&=c_{44}\frac{\partial w}{\partial x}+e_{15}\frac{\partial\phi}{\partial x},\\ \tau_{yz}&=c_{44}\frac{\partial w}{\partial y}+e_{15}\frac{\partial\phi}{\partial y},\end{aligned} \tag{4}$$

where c_{44} and e_{15} are the elastic modulus and piezoelectric coefficient, respectively. The electric displacements are expressed as

$$\begin{aligned}D_x&=e_{15}\frac{\partial w}{\partial x}-\varepsilon_{11}\frac{\partial\phi}{\partial x},\\ D_y&=e_{15}\frac{\partial w}{\partial y}-\varepsilon_{11}\frac{\partial\phi}{\partial y},\end{aligned} \tag{5}$$

where ε_{11} is the permittivity. The equations of motion for a piezoelectric medium are given as

$$\begin{aligned}\sigma_{ij,j}&=\rho\ddot{u}_i,\\ D_{i,i}&=0,\end{aligned} \tag{6}$$

where ρ is the mass density. Substituting (1)-(5) into (6) we can obtain

$$c_{44}\nabla^2 w+e_{15}\nabla^2\phi=\rho\frac{\partial^2 w}{\partial t^2}, \tag{7}$$

$$e_{15}\nabla^2 w-\varepsilon_{11}\nabla^2\phi=0, \tag{8}$$

where ∇^2 is the two-dimensional Laplacian . Let

$$\varphi=\phi-\frac{e_{15}}{\varepsilon_{11}}w, \tag{9}$$

1-4244-0647-1/07/$20.00 ©2007 IEEE

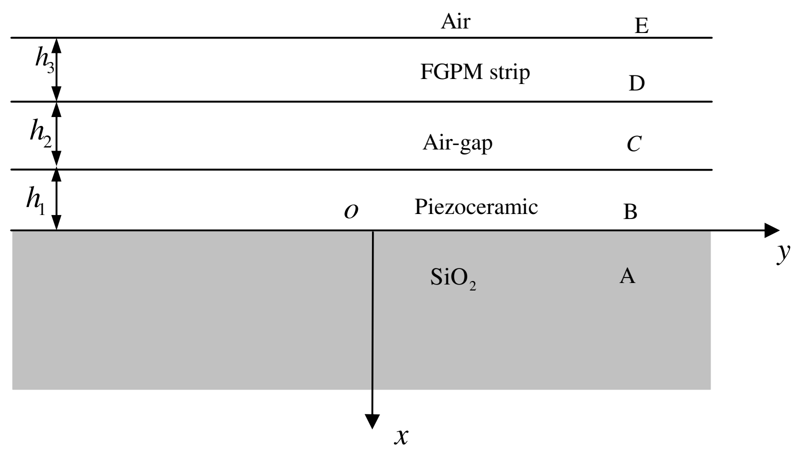

Fig. 1 Piezoelectric layered structures

equations (7)-(8) can be rewritten as

$$c_{44}^{*}\nabla^2 w=\rho\frac{\partial^2 w}{\partial t^2}\ , \tag{10}$$

$$\nabla^2\varphi=0\ , \tag{11}$$

where

$$c_{44}^{*}=c_{44}+\frac{e_{15}^2}{\varepsilon_{11}}\ . \tag{12}$$

The different regions are indicated as

$A: x\ge 0, -\infty< y<+\infty$, SiO_2.

$B: -h_1\le x\le 0, -\infty< y<+\infty$, Piezoceramic.

$C: -(h_1+h_2)\le x\le -h_1, -\infty< y<+\infty$, Air-gap.

$D: -(h_1+h_2+h_3)\le x\le -(h_1+h_2),\ -\infty< y<+\infty$, FGPM strip.

$E: x\le -(h_1+h_2+h_3), -\infty< y<+\infty$, Air.

Ⅲ. SOLUTION OF THE PROBLEM

3.1 Solutions in the region A (SiO_2)

The displacement and electric potential must satisfied the radiation condition $w_A, \phi_A\to 0$ if $x\to+\infty$. The displacement and the electric potential can be expressed as

$$\begin{aligned}w_A(x,y,t)&=A_1 e^{-kb_A x}\cos(ky-\omega t),\\ \phi_A(x,y,t)&=A_2 e^{-kx}\cos(ky-\omega t),\end{aligned} \tag{15}$$

where A_1, A_2 are unknown constants. Substituting (15) into (11) we can obtain

$$b_A^2=1-\frac{c^2}{c_{shA}^2}\ , \tag{16}$$

where c is the phase velocity and $c=\omega/k$. c_{shA} is the shear wave velocity in the SiO_2 , and

$$c_{shA}=\sqrt{c_A/\rho_A}\ . \tag{17}$$

Substituting (15) and (16) into (4) and (5) we can obtain

$$\tau_{xz}^{A}=-A_1 c_A k b_A e^{-kb_A x}\cos(ky-\omega t), \tag{18}$$

$$D_x^{A}=A_2 k\varepsilon_A e^{-kx}\cos(ky-\omega t). \tag{19}$$

3.2 Solutions in the region B (piezoceramic)

The displacement and electric potential can be expressed as (14)

$$w_B(x,y,t)=B_1 e^{-kb_B x}\cos(ky-\omega t)\ , \tag{20}$$

$$\varphi_B(x,y,t)=B_2 e^{-kx}\cos(ky-\omega t)\ . \tag{21}$$

Substituting (20) into (10) we can obtain

$$b_B^2=1-\frac{\rho_B c^2}{c_B^{*}}\ . \tag{22}$$

Substituting (20) and (21) into (4) and (5) leads to

$$\phi_B=(B_2 e^{-kx}+B_1\frac{e_B}{\varepsilon_B}e^{-kb_B x})\cos(ky-\omega t)\ , \tag{23}$$

$$\begin{aligned}\tau_{xz}^{B}=&-k[Cc_B^{*}b_B e^{-kb_B x}+De_B e^{-kx}]\\ &\cos(ky-\omega t)\end{aligned}\ , \tag{24}$$

$$D_x^{B}=B_2 k\varepsilon_B e^{-kx}\cos(ky-\omega t)\ . \tag{25}$$

3.3 Solutions in the region C(air-gap)

The electric potential should be satisfied the Laplace equation in the air-gap. The solution for the electric potential can be written as

$$\begin{aligned}\phi_C(x,y,t)=&[C_1\sinh(-kx)+C_2\cosh(-kx)]\\ &\cos(ky-\omega t),\end{aligned} \tag{26}$$

The electric displacement can be given as

$$\begin{aligned}D_x^{C}=&k\varepsilon_C[C_1\cosh(-kx)+C_2\sinh(-kx)]\\ &\cos(ky-\omega t),\end{aligned} \tag{27}$$

where ε_C is the permittivity in the vacuum.

3.4 Solutions for the region D(piezoceramics)

The constitutive equations of the functionally gradient piezoelectric materials can be written as

$$\begin{aligned}
\tau_{yz}^{\mathrm{D}} &= c_{\mathrm{D}}(x)\frac{\partial w}{\partial y}+e_{\mathrm{D}}(x)\frac{\partial \phi}{\partial y},\\
\tau_{zx}^{\mathrm{D}} &= c_{\mathrm{D}}(x)\frac{\partial w}{\partial x}+e_{\mathrm{D}}(x)\frac{\partial \phi}{\partial x},\\
D_{x}^{\mathrm{D}} &= e_{\mathrm{D}}(x)\frac{\partial w}{\partial x}-\varepsilon_{\mathrm{D}}(x)\frac{\partial \phi}{\partial x},\\
D_{y}^{\mathrm{D}} &= e_{\mathrm{D}}(x)\frac{\partial w}{\partial y}-\varepsilon_{\mathrm{D}}(x)\frac{\partial \phi}{\partial x}.
\end{aligned} \tag{28}$$

In order to obtain an exact solution, we here assume that all material coefficients of the piezoelectric strip as Fig.1 have the same exponential function distribution along the x-axis direction, i.e.

$$\begin{aligned}
&c_{\mathrm{D}}(x)=c_{\mathrm{D}}^{0}\mathrm{e}^{\beta x},\quad e_{\mathrm{D}}(x)=e_{\mathrm{D}}^{0}\mathrm{e}^{\beta x},\\
&\varepsilon_{\mathrm{D}}(x)=\varepsilon_{\mathrm{D}}^{0}\mathrm{e}^{\beta x},\quad \rho_{\mathrm{D}}(x)=\rho_{\mathrm{D}}^{0}\mathrm{e}^{\beta x}.
\end{aligned} \tag{29}$$

Substituting equations (28)-(29) into equation (7)-(8), the governing equations of the piezoelectric layer are obtained as follows

$$e_{\mathrm{D}}^{0}\beta\frac{\partial w_{\mathrm{D}}}{\partial x}+e_{\mathrm{D}}^{0}\nabla^{2}w_{\mathrm{D}}=\varepsilon_{\mathrm{D}}^{0}\beta\frac{\partial \phi_{\mathrm{D}}}{\partial x}+\varepsilon_{\mathrm{D}}^{0}\nabla^{2}\phi_{\mathrm{D}}, \tag{30}$$

$$\begin{aligned}
&c_{\mathrm{D}}^{0}\beta\frac{\partial w_{\mathrm{D}}}{\partial x}+c_{\mathrm{D}}^{0}\nabla^{2}w_{\mathrm{D}}+e_{\mathrm{D}}^{0}\beta\frac{\partial \phi_{\mathrm{D}}}{\partial x}+e_{\mathrm{D}}^{0}\nabla^{2}\phi_{\mathrm{D}}\\
&\qquad=\rho_{\mathrm{D}}^{0}\frac{\partial^{2}w_{\mathrm{D}}}{\partial t^{2}}.
\end{aligned} \tag{31}$$

By assuming

$$\psi_{\mathrm{D}}=\phi_{\mathrm{D}}-\frac{e_{\mathrm{D}}^{0}}{\varepsilon_{\mathrm{D}}^{0}}w_{\mathrm{D}},$$

equations (9) and (10) can be rewritten as follows

$$\frac{1}{\rho_{\mathrm{D}}^{0}}\left(c_{\mathrm{D}}^{0}+\frac{e_{\mathrm{D}}^{0\,2}}{\varepsilon_{\mathrm{D}}^{0}}\right)\left(\beta\frac{\partial w_{\mathrm{D}}}{\partial x}+\nabla^{2}w_{\mathrm{D}}\right)=\frac{\partial^{2}w_{\mathrm{D}}}{\partial t^{2}}, \tag{32}$$

$$\beta\frac{\partial \psi_{\mathrm{D}}}{\partial x}+\nabla^{2}\psi_{\mathrm{D}}=0. \tag{33}$$

Assuming the solutions of the equation (32) and (33) are

$$\begin{aligned}
\psi_{\mathrm{D}}&=\psi_{\mathrm{D}}(x)\exp\left[ik(y-ct)\right],\\
w_{\mathrm{D}}&=w_{\mathrm{D}}(x)\exp\left[ik(y-ct)\right].
\end{aligned} \tag{34}$$

The ordinary-differential equations can be obtained as follows

$$\psi_{\mathrm{D}}''(x)+\beta\psi_{\mathrm{D}}'(x)-k^{2}\psi_{\mathrm{D}}(x)=0, \tag{35}$$

$$w_{\mathrm{D}}''(x)+\beta w_{\mathrm{D}}'(x)+\left(\frac{c^{2}}{c_{sh}^{2}}-1\right)k^{2}w_{\mathrm{D}}(x)=0, \tag{36}$$

where $c_{sh}^{2}=\frac{1}{\rho_{\mathrm{D}}^{0}}\left(c_{\mathrm{D}}^{0}+\frac{e_{\mathrm{D}}^{0\,2}}{\varepsilon_{\mathrm{D}}^{0}}\right)$.

The solution of the equation (35) is

$$\psi_{\mathrm{D}}(x)=\mathrm{D}_{1}e^{r_{1}x}+\mathrm{D}_{2}e^{r_{2}x}, \tag{37}$$

where $r_{1,2}=\frac{-\beta\pm\sqrt{\beta^{2}+4k^{2}}}{2}$.

The solution of the equation (36) is obtained as

$$\begin{aligned}
w_{\mathrm{D}}(x)&=(\mathrm{D}_{3}e^{sx}\cos\lambda x+\mathrm{D}_{4}e^{sx}\sin\lambda x)\\
&\qquad\exp\left[ik(y-ct)\right],
\end{aligned} \tag{38}$$

when $c^{2}>c_{sh}^{2}\left(1+\frac{\beta^{2}}{4k^{2}}\right)$.

where

$$s=-\beta/2,\ \lambda=\frac{1}{2}\sqrt{4k^{2}\left(1-\frac{c^{2}}{c_{sh}^{2}}\right)-\beta^{2}}.$$

Substituting equation (37) and equation (38) into equation (30), the electric potential can be obtained

$$\begin{aligned}
\phi_{\mathrm{D}}&=(\mathrm{D}_{1}e^{r_{1}x}+\mathrm{D}_{2}e^{r_{2}x})\exp\left[ik(y-ct)\right]\\
&+\frac{e_{\mathrm{D}}^{0}}{\varepsilon_{\mathrm{D}}^{0}}\left(\mathrm{D}_{3}e^{sx}\cos\lambda x+\mathrm{D}_{4}e^{sx}\sin\lambda x\right)\exp\left[ik(y-ct)\right].
\end{aligned} \tag{39}$$

when $c^{2}>c_{sh}^{2}\left(1+\frac{\beta^{2}}{4k^{2}}\right)$.

$$\begin{aligned}
\tau_{yz}^{\mathrm{D}}&=c_{\mathrm{D}}^{0}e^{(\beta+s)x}ik(\mathrm{D}_{3}\cos\lambda x+\mathrm{D}_{4}\sin\lambda x)\exp[ik(y-ct)]\\
&+e_{\mathrm{D}}^{0}e^{\beta x}ik[\mathrm{D}_{1}e^{r_{1}x}+\mathrm{D}_{2}e^{r_{2}x}+\\
&\frac{e_{\mathrm{D}}^{0}}{\varepsilon_{\mathrm{D}}^{0}}e^{sx}(\mathrm{D}_{3}\cos\lambda x+\mathrm{D}_{4}\sin\lambda x)]\exp[ik(y-ct)],
\end{aligned} \tag{40}$$

$$D_{x}^{\mathrm{D}}=-\varepsilon_{\mathrm{D}}^{0}e^{\beta x}(\mathrm{D}_{1}r_{1}e^{r_{1}x}+\mathrm{D}_{2}r_{2}e^{r_{2}x})\exp[ik(y-ct)]. \tag{41}$$

3.5 Solutions in the region E(air)

Electric potential must be satisfied the Laplace equation in the air. Let

$$\phi_{\mathrm{E}}(x,y,t)=E\mathrm{e}^{kx}\cos(ky-\omega t), \tag{42}$$

$$D_{x}^{\mathrm{E}}=-Ek\varepsilon_{\mathrm{E}}\mathrm{e}^{kx}\cos(ky-\omega t). \tag{43}$$

Ⅳ. BOUNDARY CONDITIONS

The continuous conditions and boundary conditions for the electrically open case on the surface of the strip are given as follows

At $x=0$:

$$\phi_{\mathrm{A}}=\phi_{\mathrm{B}},\ w_{\mathrm{A}}=w_{\mathrm{B}},\ \tau_{xz}^{\mathrm{A}}=\tau_{xz}^{\mathrm{B}},\ D_{x}^{\mathrm{A}}=D_{x}^{\mathrm{B}}. \tag{36}$$

At $x=-h_{1}$: $\phi_{\mathrm{B}}=\phi_{\mathrm{C}},\ D_{x}^{\mathrm{B}}=D_{x}^{\mathrm{C}}$. (37)

At $x=-(h_{1}+h_{2})$: $\phi_{\mathrm{C}}=\phi_{\mathrm{D}},\ D_{x}^{\mathrm{C}}=D_{x}^{\mathrm{D}}$. (38)

At $x=-(h_{1}+h_{2}+h_{3})$:

$$\phi_{\mathrm{D}}=\phi_{\mathrm{E}},\ D_{x}^{\mathrm{D}}=D_{x}^{\mathrm{E}},\ \tau_{xz}^{\mathrm{D}}=0 \tag{39}$$

Substituting (15)-(35) into (36)-(40) we can obtain the constants $A_{1},A_{2},B_{1},B_{2},C_{1},C_{2},D_{1},D_{2},D_{3},D_{4},E$ for electrically open case.

Ⅴ. RESULTS AND DISCUSSION

The material constants are given as Table 1. The calculation results of the phase velocity and the distribution of the displacement and shear stress are illustrated as Figs. 2-4.

Fig.2 shows that the phase velocity for the electrically open case with different gradient factor with $h_2 = 0.0001mm$. We can find that the phase velocity increases with the increases of the gradient factor under the same non-dimensional wavenumber.

Fig.3 indicates that the distribution of the displcement along the thickness of the FGPM strip for the electrically open case. It can be seen that the small difference among the displacements for the same thickness coordinate along the FGPM strip due to the variant gradient factor.

Fig.4 shows that the distribution of the stear stress along the thickness of the FGPM strip for the electrically open case. We can find that the stress decreases with the increase of the gradient factor.

Ⅵ. CONCLUSIONS

Shear horizontal gap wave propagating between functionally graded piezoelectric material (FGPM) layer and a layered piezoelectric structure is investigated analytically. The electrically open conditions on strip surface are applied to solve this problem. The phase velocity can be numerically calculated for the electrically open case, with different thickness of the layer and wavenumber. The effect of the gradient variation about material on the phase velocity is discussed in detail. We find that gradient distributing of the material properties has remarkable effect on the phase velocity of the gap waves.

Acknowledgements

This work is supported by the Key Industrial Project of Ningbo Bureau of Science and Technology (No.2005B100015) and a grant from the Science and Technology Division, Zhejiang Province, China, under Key Technological Initiative Program (Project No. 2006C14021).

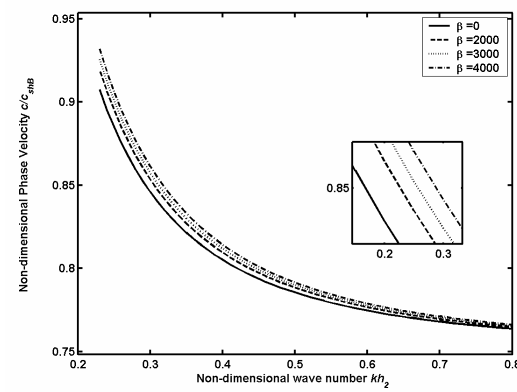

Fig.2 Phase velocity versus non-dimensional wavenumber with $h_2 = 0.0001mm$

Table 1 Constants of the Materials

	$\rho(10^3 kg/m^3)$	$e_{15}(C/m^2)$	$\varepsilon_{11}(10^{-9} F/m)$	$c_{44}(10^9 N/m^2)$
SiO_2(region A)	2.2	0	0.033	31.2
$BaTiO_3$(region B)	5.7	11.4	9.872	44
FGPM	$\rho_D^0 = 7.5$	$e_{15}^0 = 17$	$\varepsilon_{11}^0 = 15.1$	$c_{44}^0 = 23.1$

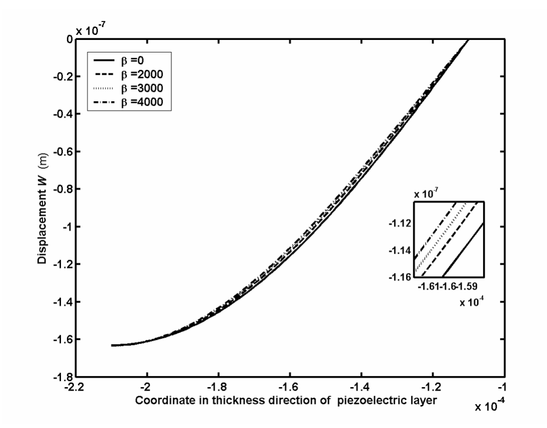

Fig.3 Distribution of the displacement along the thickness of the FGPM strip with $h_2 = 0.0001mm$

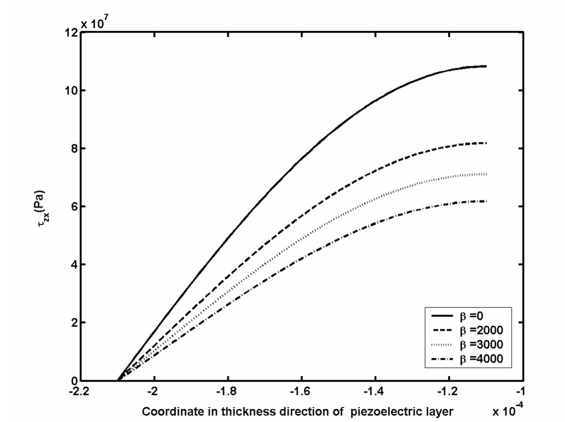

Fig.4 Distribution of the shear stress along the thickness of the FGPM strip with $h_2 = 0.0001mm$

References

[1] Gulyaev, Y.V. and Plesskii, V.P., “Acoustic gap waves in piezoelectric materials,” Sov. Phys. Acoust. Vol. 23, pp. 410-413, 1977.

[2] Yang, J.S. and Zhou, H.G., “Propagation and amplification of gap waves between a piezoelectric half-space and a semiconductor film,” Acta Mech. Vol. 176, pp. 83-93, 2005.

[3] Li, X.F. and Yang, J.S., “Piezoelectric gap waves between a piezoceramic half-space and a piezoceramic plate,” Sensors and Actuators A, doi:10.1016/j.sna. 2006.02. 0412006

[4] White, R.M. et al., Direct piezoelectric coupling to surface elastic waves. Appl. Phys. Lett. 7, 314, 1965

[5] Liu, G.R., and Tani, J., Surface waves in functionally gradient piezoelectric plates, Journal of Vibration and Acoustics, 116, 440-448, 1994

[6] Liu, G.R., Tani, J., Characteristic of wave propagation in functionally gradient piezoelectric material plates and its response analysis. Part 1: theory; Part 2: calculation results, Transactions of the Japan Society of Mechanical Engineers, 57A (541), 2122-2133

[7] Liu, G.R., Tani, J., SH surface waves in functionally gradient piezoelectric material plates. Transactions of the Japan Society of Mechanical Engineers, 58A (547), 504-507

[8] Liu, G.R., Tani, J., Surface waves in functionally gradient piezoelectric plates. Transactions of the American Society of Mechanical Engineers, 116, 440-448, 1994

[9] Liu, G.R., Tani, J., Ohyoshi, T., Lamb waves in a functionally gradient material plates and its transient response. Part 1: theory; Part 2: calculation results. Transactions of the Japan Society of Mechanical Engineers 57 (535), 131-142, 1991

[10] Liu, G.R., Tani, J., Ohyoshi, T., Watanabe, K., Characteristic wave surface in anisotropic laminated plates. Journal of Vibration and Acoustics, 113, 279-285, 1991

[11] Han, X., Liu, G.R., and Lam, K.Y., Ohyoshi, T., A quadratic element for analyzing stress waves in functionally graded materials and its applications for material characterization. Journal of Sound and Vibration, 236(2), 307-321, 2001

[12] Han, X., Liu, G.R., Xi, Z.C., Lam, K.Y., Characteristics of waves in a functionally graded cylinder. International Journal for Numerical Methods in Engineering, 53(3), 653-676, 2002

[13] Han, X., and Liu, G.R., Elastic waves propagating in a functionally graded piezoelectric cylinder. Smart Materials and Structures, 12(6), 962-971, 2003

[14] Li, X.Y., Wang, Z.K., and Huang, S.H., Love waves in functionally graded piezoelectric materials. International Journal of Solids and Structures, 41, 7309-7328, 2004